Platform design and product quality*

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Abstract

Product quality is essential for consumer surplus and social welfare. We study how incentives for product quality are shaped by platform fees and quality-based steering. A profit-maximizing platform rewards quality and charges a commission rather than using a transaction fee. Steering partly offsets the commissions' adverse quality incentives, but quality remains inefficiently low. Reducing quality rewards can nevertheless improve welfare by expanding quantity. Commission caps have a non-monotonic effect on welfare: moderate caps increase quality and welfare, while tighter caps reduce welfare despite efficient quality. The results show why neither stronger quality incentives nor stricter commission regulation necessarily improves welfare.

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JEL Classification Codes: D47, D83, L81

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1 Introduction

Digital platforms influence both which products consumers see and how much revenue sellers retain from a sale. These two choices jointly shape sellers' incentives to provide quality. A higher-quality product may attract more consumer attention, but the seller bears its full production cost while sharing the resulting revenue with the platform. We study whether a platform's steering algorithm and fee structure induce socially desirable quality choices, and how interventions in either instrument affect welfare.

Consider a monitor sold through an online marketplace. A higher resolution increases consumers' willingness to pay, but also raises the marginal cost of production. If the platform presents higher-resolution monitors more prominently, it creates an additional incentive to provide quality. A commission works in the opposite direction by reducing the revenue that sellers retain. This example captures the mechanism of our model: the platform can use visibility to offset the quality distortion created by its own fees, but chooses both instruments to maximize fee revenue. Although the resulting quality is below the efficient one, more steering towards higher quality would further reduce welfare.

Consumer attention is central to this mechanism. Online marketplaces offer more products than consumers can assess, so presentation affects which products enter their consideration sets. Sellers therefore compete not only for sales conditional on being considered, but also for the opportunity to reach consumers. A substantial literature studies how intermediaries use prominence and recommendations to influence prices, matching, and advertising (e.g., Armstrong and Zhou, 2011, 2022; Athey and Ellison, 2011; Chen and He, 2011; Dinerstein et al., 2018; Eliaz and Spiegler, 2011; Hagiu and Jullien, 2011; Heidhues et al., 2023; Johnson et al., 2023; Teh and Wright, 2022). Recommendations and fees also shape sellers' returns to improving their products, affecting both consumer value and production costs. Understanding their joint effect on quality is therefore essential for evaluating platform design and regulation. We study this interaction by allowing sellers to choose quality in response to both instruments.

We develop a simple model with a monopolistic platform and a fixed number of initially identical firms. The platform commits to a commission, or value fee, charged as a proportion of sales revenue; a transaction fee charged per unit sold; and a quality-reward parameter. Firms then choose prices and qualities. The algorithm allocates consumer awareness through a Tullock contest success function: a high exponent makes awareness more sensitive to relative quality. Conditional on awareness, demand depends on the firm's own price and quality. Quality raises

willingness to pay and marginal production costs. By isolating competition for attention, the model makes the interaction between steering, fees, and endogenous quality tractable.[1] Its responsive quantity margin also lets us explain why improving quality need not improve welfare, particularly when regulation induces the platform to adjust its other instruments.

We first characterize firms' behavior for any platform design. Whenever positive trade is profitable, the game between sellers has a unique Nash equilibrium, which is interior and symmetric. Anticipating this equilibrium, the platform optimally chooses a positive commission and a positive quality reward, but no transaction fee. A monotonicity condition on relative cost and benefit elasticities ensures uniqueness of the platform optimum.

The commission weakens quality incentives because sellers bear the full cost of better quality but retain only part of its revenue benefit. The platform partly offsets this distortion by rewarding quality with greater visibility. However, fully restoring efficient quality is less profitable than retaining a higher commission and accepting some quality under-provision. A positive transaction fee is also unattractive: the platform can replace it with a higher commission and stronger quality rewards, preserving quality, price, and quantity while transferring profits from sellers to itself. This result relates to the comparison between ad valorem and unit taxation under monopoly (Skeath and Trandel, 1994), but here the platform can also adjust sellers' incentives through steering.

Our second result distinguishes quality under-provision from insufficient steering. Starting from the platform optimum with quality under-provision, a small reduction in the quality reward, holding fees fixed, lowers quality further but increases quantity, and as a consequence, increases consumer surplus, aggregate firm profits, and total welfare. The expansion in consumption outweighs the loss from lower quality. Thus, quality can be too low even though the incentive to compete for visibility is locally excessive. Commissions are excessive in the same sense: A small commission reduction, holding steering and the transaction fee fixed, also raises quality, quantity, and welfare when the elasticity of production costs with respect to quality is locally nondecreasing. These are, however, local changes in one instrument, not predictions about regulation that permits the platform to adjust its other choices.

We therefore study commission caps separately, allowing the platform to re-optimize both steering and the transaction fee. Caps above the unconstrained commission do not affect the outcome. Moderately binding caps increase quality and quantity while the platform continues to rely exclusively on commission revenue.

[1] Unlike Shaked and Sutton (1982), we do not study quality differentiation as a means of softening price competition.

Welfare rises as the cap is tightened, until quality reaches the first-best level. Below this threshold for the commission cap, the platform introduces a positive transaction fee and reduces steering. Quality remains efficient, but quantity and welfare fall as the cap is tightened further. Welfare is consequently maximized at an intermediary commission cap, which just induces first-best quality.

A numerical example illustrates both the local effects and the response to regulation. With benefit $B(Q) = 10\sqrt{Q}$ and production cost $C(Q) = Q$, the unconstrained commission is one third, whereas the welfare-maximizing cap is lower. A zero commission cap yields lower welfare than the unconstrained outcome, despite restoring efficient quality. The policy lesson is that restrictions on one fee must account for substitution toward other instruments, and that efficient quality alone is not a sufficient measure of regulatory success.

Related Literature. Our contribution connects two strands of literature: the design of steering and monetization, and the effect of intermediation on sellers' quality choices. In the steering literature, platforms trade off consumer matching, seller competition, and fee revenue (e.g., Armstrong and Zhou, 2022; Dinerstein et al., 2018; Heidhues et al., 2023; Nocke and Rey, 2024; Teh, 2022; Teh and Wright, 2022). Closely related, Bar-Isaac and Shelegia (2025) jointly study ad valorem fees and steering through either algorithms or advertising auctions. Their competitive benchmark permits extraction of monopoly profits under either method, but the optimal fee depends on how steering is implemented. We likewise analyze steering and monetization jointly, while focusing on endogenous quality and the distinction between local welfare effects and commission regulation.

Johnson et al. (2023) study rules that reward price cuts with additional exposure and can disrupt collusion, including when sellers use pricing algorithms. Ng (2025) studies competing platforms whose recommendations rank products by value for money. More precise recommendations intensify price competition and screen out lower-quality firms, whose qualities are drawn exogenously. In a different setting, Peitz and Sobolev (2026) show that competition between firms choosing prices and purchase recommendations can induce inefficient recommendation biases. These papers examine pricing, selection, or information provision. By isolating how quality-dependent visibility changes firms' production choices, our contribution is to this literature is to endogenize quality.

Belleflamme and Peitz (2010) study the effect of intermediation on seller investments, including quality improvements. These affect indirect network effects and the access fees subsequently chosen by competing platforms. Depending on the investment and participation patterns, for-profit intermediation can turn underinvestment

into overinvestment. In our model, the platform instead commits to steering and sales-based fees before firms choose quality. Aridor and Gonçalves (2022) also endogenize quality investment, but study a platform that both produces content and recommends products. Investment affects the probability of high quality, and recommendations convey information about realized quality. Self-preferencing can reduce investment and consumer welfare. Our mechanism operates without platform production or self-preferencing: quality distortions arise from fees and the allocation of awareness among independent sellers.

Bergemann and Bonatti (2024) study firms choosing product quality for consumers on and off a platform. Personalized offers on the platform feature efficient quality and matching, while off-platform quality can be distorted downwards. Their mechanism involves consumer incentive-compatibility constraints between purchasing on and off the platform, which we do not model. Another difference is that firms pay marketing budgets to the platform, rather than the proportional sales fee that weakens quality incentives in our setting.

Casner and Teh (2025) study how discovery and membership services affect creators' content choices. Creators trade off broader appeal against higher revenue per viewer from niche content, linking recommendations and monetization to product design.[2] Their focus is on platform business models and the interaction between advertising and membership revenues. Our quality choice instead raises both willingness to pay and production costs, and we jointly optimize quality rewards and two sales-based fees.

In closely related work in progress, Guivernau and Shelegia (2026) study a monopoly platform whose recommendations affect firms' prices and costly quality choices. Their Tullock recommendation score combines consumer net utility and platform fee revenue. They examine how algorithmic alignment and precision affect outcomes when the platform chooses an ad valorem fee. We instead endogenize the quality-reward parameter jointly with commission and transaction fees. Moreover, their consumers have unit demand, whereas quantity conditional on awareness responds to price and quality in our model. This quantity margin is central to our welfare results: lower quality can accompany higher welfare, and efficient quality under a commission cap need not imply efficient consumption.

Finally, Gao et al. (2023) use a complexity theory approach in which firms allocate resources to fees and quality, and the platform rewards fees through steering. We derive demand from consumer utility maximization, let sellers choose prices,

[2]Their earlier working paper (Casner and Teh, 2022) uses a Tullock recommendation contest with an endogenous sensitivity parameter, providing a close methodological connection. The choice of a contest exponent is also studied in a different context by Sahm (2022).

and endogenize the contest exponent. These differences allow us to trace the joint adjustment of quality, prices, and quantities when platform instruments change.

Section 2 presents the model and characterizes the seller equilibrium. Section 3 solves the platform problem, derives the local welfare effects, and analyzes commission caps. Section 4 discusses quality investment costs, more general demand, and larger consideration sets, identifying additional mechanisms and limits to the baseline analysis. Section 5 concludes.

2 Model

2.1 Setup

There are $n \geq 2$ firms. Each firm $i \in N = \{1, 2, \ldots, n\}$ chooses a quality $Q_i \geq 0$ and a price $p_i \geq 0$ for its product. The marginal costs of production $C(Q_i)$ are increasing and weakly convex in quality; i.e., $C'(Q_i) > 0$ and $C''(Q_i) \geq 0$.

A unit mass of consumers considers each product i with probability $A(Q_i, Q_{-i})$ (that results from the steering algorithm) and then demands $D(p_i, Q_i) = B(Q_i) - p_i$ of it. The benefit function $B(Q_i)$ is assumed to be increasing and concave in quality; i.e., $B'(Q_i) > 0$ and $B''(Q_i) < 0$. Moreover, B and C are continuous at zero, twice continuously differentiable for $Q > 0$, $B(0) = C(0) = 0$, $B'(0+) > C'(0+)$ (allowing $B'(0+) = +\infty$), and $B'(Q) < C'(Q)$ for sufficiently large Q.

A (monopolistic) platform chooses an algorithm that can reward quality choices by making high quality products more prominent. In particular, it chooses parameter $\rho \geq 0$ in the following Tullock-like function: $A(Q_i, Q_{-i}) = \frac{Q_i^\rho}{\sum_{k \in N} Q_k^\rho}$, the algorithm. Observe that for $\rho = 0$, we have $A(Q_i, Q_{-i}) = \frac{1}{n}$; which we define to also hold for any $\rho \geq 0$ when $\sum_{k \in N} Q_k = 0$. A literal interpretation of the model is that $A(Q_i, Q_{-i})$ is the share of consumers who are made aware of product i and that they only consider this product.[3]

Moreover, the platform defines the fee structure by setting a commission, called value fee $\tau \geq 0$, and a per-unit fee, called transaction fee $\phi \geq 0$. Hence, firm i's profits are given by $\pi_i = A(Q_i, Q_{-i}) \cdot D(p_i, Q_i) \cdot [p_i(1 - \tau) - C(Q_i) - \phi]$ and the platform profits are $\Pi^{Pl} = \sum_i A(Q_i, Q_{-i}) \cdot [\tau p_i + \phi] \cdot D(p_i, Q_i)$.

The timing of the model is as follows:

1. The platform chooses the algorithm's quality-reward parameter ρ, the value fee τ and the transaction fee ϕ.

2. The n firms choose their quality Q_i and price p_i (respectively, quantity y_i).

[3] More general interpretations are discussed in Section 4.3 below.

3. Consumers consider products according to the algorithm $A(Q_i, Q_{-i})$ and choose how much to buy according to their demand function $D(p_i, Q_i)$.

In Stage 1, the platform sets the parameters. In Stage 2, the firms play a simultaneous game. In stage 3, consumer behavior generates the payoffs for the firms and the platform.

2.2 Benchmark: First best

Overall welfare is the weighted sum of welfare in each market, while the weight for market i is given by $A(Q_i, Q_{-i})$. In each market, welfare consists of consumer surplus, producer surplus and platform surplus, where the latter two are simply their respective profits. Consumer surplus in a market with price p_i, corresponding quantity y_i, and quality Q_i is $CS = \int_0^{y_i} (P(q, Q_i) - p_i) dq$, where $P(y, Q) := B(Q) - y$ is the inverse demand function. (In this inverse demand formulation it becomes apparent that the benefit function $B(Q)$ can be interpreted as the willingness-to-pay.)

Consider a social planner that can impose all decisions made by the platform and the firms. In every market, it will impose price equal to marginal cost, and the optimal quality $\hat{Q} := \arg\max_Q \int_0^y P(q, Q) dq - C(Q)y$. Our assumptions assure that this maximization over quality has a unique solution, which is characterized by the first order condition $B'(\hat{Q}) = C'(\hat{Q})$. The algorithm that allocates attention across markets with quality-reward parameter ρ then does not matter, as any market generates the same first best welfare.

Remark 1 (First best)**.** *The socially optimal choices of the platform and the firms are attained for:* $\hat{\tau} = \hat{\phi} = 0$, $\hat{p} = C(\hat{Q})$ *(and hence* $\hat{y} = B(\hat{Q}) - C(\hat{Q})$*) and*

$$B'(\hat{Q}) = C'(\hat{Q}). \tag{1}$$

Zero fees and prices equal to marginal costs assure that there is no welfare loss due to pricing. More importantly, the social planner would choose quality at its optimal level, where the marginal benefit of quality, which is the average increase in willingness-to-pay due to higher quality, equals the marginal costs of quality, as expressed by (1). This level of quality, $\hat{Q}$, will serve as a benchmark in the equilibrium analysis. In particular, we will call quality under-provided (over-provided) if it is below (above) $\hat{Q}$.

2.3 Firm's Problem

Each firm i solves the following problem:

$$\max_{Q_i, p_i} \quad \pi_i = A(Q_i, Q_{-i}) \cdot D(p_i, Q_i) \cdot [p_i(1-\tau) - C(Q_i) - \phi] \ .$$

Using the inverse demand instead, the problem becomes:

$$\max_{Q_i, y_i} \quad \pi_i = A(Q_i, Q_{-i}) \cdot y_i \cdot [P(y_i, Q_i)(1-\tau) - C(Q_i) - \phi] \ .$$

The interior best response of firm i to Q_{-i} satisfies two first order conditions:[4]

$$y_i = \frac{P(y_i, Q_i)(1-\tau) - C(Q_i) - \phi}{-P'(y_i, Q_i)(1-\tau)} \tag{2}$$

$$\frac{A'(Q_i, Q_{-i})}{A(Q_i, Q_{-i})} = \frac{C'(Q_i) - \frac{\partial P(y_i, Q_i)}{\partial Q_i}(1-\tau)}{P(y_i, Q_i)(1-\tau) - C(Q_i) - \phi} \tag{3}$$

The first equation has the standard interpretation that optimal monopolistic quantity decreases (respectively, price increases) with costs, with fees and depends on the price elasticity of demand. Denote the profit per unit sold as $v(y_i, Q_i) := P(y_i, Q_i)(1-\tau) - C(Q_i) - \phi$. Then the second equation reads

$$\frac{A'(Q_i, Q_{-i})}{A(Q_i, Q_{-i})} = \frac{-\frac{\partial v(y_i, Q_i)}{\partial Q_i}}{v(y_i, Q_i)}$$

and can be interpreted as follows. Firms choose quality such that the relative gain in awareness (LHS) equals the relative loss in per-unit profit (RHS). A positive quality reward therefore induces firms to choose quality above the level that maximizes their per-unit profit.

2.4 Equilibrium

Consider the game between firms in the second stage, where the platform parameters chosen in the first stage are fixed.

Proposition 1. *For any platform parameters ρ, τ, and ϕ, there is a symmetric Nash equilibrium in the game between firms (i.e., $y_i^* = y$ and $Q_i^* = Q, \forall i \in N$). If*

[4] We use the convention that the derivative notation "prime" refers to the first argument: $f'(x_1, x_2, ...) := \frac{\partial f(x_1, x_2, ...)}{\partial x_1}$.

interior (i.e., $y, Q > 0$), it is unique and satisfies:

$$\frac{C'(Q) - B'(Q)(1-\tau)}{B(Q)(1-\tau) - C(Q) - \phi} = \frac{1}{Q} \cdot \frac{\rho(n-1)}{2n}. \tag{4}$$

The proof of Proposition 1, as each proof, is relegated to the appendix. It distinguishes between two cases that depend on the platform's settings. One of them is a trivial case in which per-unit profits of firms are never positive. This yields only equilibria with boundary solutions such that every firm chooses quantity $y_i = 0$ and arbitrary quality Q_i. As a consequence, the firm profit and the platform profit is zero in this case. The other case is more relevant. It yields unique best responses ($y_i > 0$ and $Q_i > 0$ for every firm i) which satisfy the two first order conditions, and cannot be different for different firms. Therefore, in this case, there is a unique equilibrium between firms, and it is symmetric and interior. In this equilibrium firms generate positive profits, which also means positive profits for the platform (unless both fees are zero). Although the equilibrium quality is only implicitly given by Equation (4), the proposition still reveals several comparative-static effects.

For $\rho \equiv 0$, the firms choose quality such that the LHS of (4) is zero. That is obtained at $\tilde{Q}$ that satisfies $B'(\tilde{Q})(1-\tau) = C'(\tilde{Q})$. Therefore, by concavity of B and convexity of C, the equilibrium quality is decreasing in the value fee τ. Only for $\tau = 0$ (and still $\rho \equiv 0$), the first best quality $\hat{Q}$ is chosen (as $B'(\hat{Q}) = C'(\hat{Q})$).

For $\rho > 0$, quality is chosen higher than $\tilde{Q}$, as the LHS of (4) must be positive. In contrast to standard Tullock contests, there is no issue of non-existence for large exponents. As it follows from the proof of Proposition 1, the larger ρ, the closer the equilibrium quality is to the implicit upper bound, where the denominator of Eq. (4) becomes zero.

The intuition for the downwards distortion by the value fee is that investments into quality increase costs $C(Q)$ and benefits $B(Q)$ differently for different actors: Whereas the costs are fully borne by the firm, only part $(1-\tau)$ of the benefits go to the firm, the remaining part τ is captured by the platform. The intuition for the upwards distortion by the algorithm parameter ρ is rather straight-forward. Additional reward for quality, i.e., $\rho > 0$, creates a quality competition among firms for consumer attention. In the absence of a value fee, $\tau \equiv 0$, this leads to an over-provision of quality ($Q > \hat{Q}$), similar to the inefficiency of a Tullock contest, where effort is over-provided. Finally, consider the effect of the number of firms. Larger n leads to (slightly) larger Q, in contrast to standard Tullock contests, as we will discuss further below.

3 Main results

3.1 Platform-Optimal Design

In the first stage of the model, the platform operator chooses a platform design to maximize profits, anticipating the symmetric Nash equilibrium that will be played among the firms. The platform profit is given by $\sum_i \frac{Q_i^\rho}{\sum_{k\in N} Q_k^\rho} \cdot [\tau P(y_i, Q_i) + \phi] y_i$ and the unique Nash equilibrium of Stage 2 is characterized in Proposition 1. Hence, the platform problem can be written as

$$\begin{aligned} \max_{\rho,\tau,\phi} \qquad & \Pi_{Pl} = \left[\tau \frac{B(Q)(1-\tau) + C(Q) + \phi}{2(1-\tau)} + \phi\right] \frac{B(Q)(1-\tau) - C(Q) - \phi}{2(1-\tau)} \\ \text{subject to} \qquad & (4), \rho \geq 0, \tau \geq 0, \phi \geq 0. \end{aligned} \tag{$\mathcal{P}$}$$

We are now ready to state our main result.

Theorem 1. *The solution (ρ^*, τ^*, ϕ^*) to the platform's problem $\mathcal{P}$ satisfies*

$$\begin{cases} (i) \quad \rho^* > 0 \ , \\ (ii) \quad \tau^* = \sqrt{1 - \left[\frac{C'(Q)}{B'(Q)}\right]^2} \ , \\ (iii) \quad \phi^* = 0. \end{cases}$$

If, additionally, the quality elasticity $E(Q) := \frac{C'(Q)B(Q)}{B'(Q)C(Q)}$ is nondecreasing on $(0, \hat{Q})$, the solution is unique.

Theorem 1 characterizes the optimal choice by the platform. First, it chooses a strictly positive reward for quality $\rho^* > 0$, creating quality competition for consumer attention.[5] We know from the discussion of Proposition 1 that this per se leads to an upwards distortion of firm quality. At the same time, however, it chooses a positive value fee $\tau^* > 0$ that generates platform revenue and distorts firm quality downwards. Finally, it refrains from choosing a positive transaction fee ϕ^*. A positive transaction fee is dominated by a higher commission combined with stronger quality rewards. The platform can replace the transaction fee with a commission increase that preserves sellers' pricing incentives, then strengthen steering to offset the reduced quality incentives. Quality, price, and quantity remain unchanged, but profits shift from sellers to the platform.

It turns out that the upward distortion by the competition for attention and the downward distortion by the value fee have a net negative effect. This can

[5] More specifically, $\rho^* = \frac{2n}{n-1} Q^* \frac{C'(Q^*) - B'(Q^*)(1-\tau^*)}{B(Q^*)(1-\tau^*) - C(Q^*)}$.

be seen, e.g., from the second part of Theorem 1, where the optimal value fee $\tau^* = \sqrt{1 - \left[\frac{C'(Q)}{B'(Q)}\right]^2} > 0$ requires $B'(Q) > C'(Q)$ and hence, $Q < \hat{Q}$.

Corollary 1. *In equilibrium, quality is under-provided.*

The commission weakens quality incentives because sellers bear the full cost of better quality but retain only part of the additional revenue. The platform partly offsets this distortion by rewarding quality with greater visibility. However, it chooses the reward to maximize fee revenue, rather than total surplus. Fully restoring efficient quality is therefore less profitable than retaining a higher commission and accepting some quality under-provision.

3.2 Illustration and Local Welfare Effects

To illustrate the results and to have a quantification of the effects, let us study a numerical example.

Example. *Let $C(Q) = Q$ and $B(Q) = 10\sqrt{Q}$ and, where needed, $n = 2$. The first-best quality is $\hat{Q} = 25$. From Theorem 1 and Proposition 1, the unconstrained equilibrium satisfies*

$$\rho^* = 2\sqrt{2}, \qquad \tau^* = \frac{1}{3}, \qquad \phi^* = 0, \qquad Q^* = \frac{200}{9} \approx 22.22.$$

To see how the fee structure and the reward for quality distort qualities, let us first study multiple scenarios where at least two parameters are set to zero and then turn to local effects at the equilibrium values. If $\rho \equiv 0$, $\tau \equiv 0$, and $\phi \equiv 0$, we receive optimal quality $\hat{Q} = 25$ (both in the first best scenario with marginal cost pricing, $\hat{p} = 25$, and in the second best with monopolistic pricing $p = 37.5$). If $\rho \equiv 0$, $\tau \equiv 0$, and $\phi \equiv 12.5$, then the equilibrium quality is $Q = 25$. The transaction fee does not distort quality, as long as $\rho \equiv 0$. If $\rho \equiv 0$, $\tau \equiv \frac{1}{3}$ and $\phi \equiv 0$, then the equilibrium quality is $Q = 11.1$. The value fee distorts quality downwards. If $\rho \equiv 2\sqrt{2}$, $\tau \equiv 0$ and $\phi \equiv 0$, then the resulting quality is $Q = 50$. A reward for quality in terms of consumer awareness distorts quality upwards.

The following table summarizes these scenarios and provides each group's surplus for each scenario. Clearly, for consumers the first best is by far the best scenario. Letting firms choose the price, but keeping platform fees at zero (second best), reduces welfare because of monopoly pricing. The transaction fee further reduces welfare by a form of double marginalization problem (where the platform's fee and the firm's price exceed their marginal costs). Setting a value fee instead harms welfare less.

Table 1: Numerical Example. Quality and surpluses in different scenarios, where at least two of the platform parameters—quality-reward parameter ρ, value fee τ, and transaction fee ϕ—are set to zero, compared to equilibrium.

	Quality Q	**Welfare**	**Platform profit**	**Consumer surplus**	**Firm profit**	**Price** p	**Quantity** y
first best: $\hat{\rho} \equiv 0$, $\hat{\tau} \equiv 0$, $\hat{\phi} \equiv 0$, $\hat{p} \equiv 25$	25.0	312.5	0	312.5	0	25	25
second best: $\hat{\rho} \equiv 0$, $\hat{\tau} \equiv 0$, $\hat{\phi} \equiv 0$	25.0	234.38	0	78.13	156.25	37.50	12.50
only transaction fee: $\rho \equiv 0$, $\tau \equiv 0$, $\phi \equiv 12.5$	25.0	136.72	78.13	19.53	39.06	43.75	6.25
only value fee: $\rho \equiv 0$, $\phi \equiv 0$, $\tau \equiv \frac{1}{3}$	11.1	150.46	69.44	34.72	46.30	25	8.33
only reward for quality: $\rho \equiv 2\sqrt{2}$, $\tau \equiv 0$, $\phi \equiv 0$	50.0	160.85	0	53.62	107.23	60.36	10.36
equilibrium: $\rho^* = 2\sqrt{2}$, $\phi^* = 0$, $\tau^* = \frac{1}{3}$	22.2	148.19	92.59	23.83	31.77	40.24	6.90

We now turn to local welfare effects. Figure 1 illustrates how quality and surpluses respond to different quality-reward parameters ρ and different value fees τ, keeping the other parameters at their equilibrium value. Observe that both firms and consumers preferred the fee and the quality-reward parameter to be zero, but the platform chooses them higher. This choice is excessive in the sense that reducing the parameter ρ, or reducing the value fee τ, would increase overall welfare.[6]

As the example illustrates, under-provision of quality does not imply that the quality-reward parameter ρ is set too low in equilibrium. Let us now study how this insight from the example generalizes to our model. For that purpose let us slightly reduce ρ, respectively τ, keeping all other parameters fixed at their equilibrium value.

Proposition 2. *Starting from the platform-optimal design characterized by Theorem 1:*

(i) A sufficiently small reduction in the quality-reward parameter ρ, holding $\tau = \tau^*$ and $\phi = \phi^* = 0$ fixed, reduces equilibrium quality and increases quantity, consumer surplus, aggregate firm profits, and social welfare.

(ii) Suppose the elasticity of production costs with respect to quality, $\varepsilon_C(Q) := \frac{QC'(Q)}{C(Q)}$, satisfies $\varepsilon_C'(Q^*) \geq 0$. A sufficiently small reduction in the value fee τ, holding $\rho = \rho^*$ and $\phi = \phi^* = 0$ fixed, increases equilibrium quality, quantity, consumer surplus, aggregate firm profits, and social welfare.

[6] Consumer surplus can also enter platform objectives; see, e.g., Dinerstein et al. (2018); Xu and Yang (2026). Here the local result shows that reducing steering from the profit optimum raises consumer surplus at zero first-order cost to platform profit; it does not characterize the global optimum of a platform with a different objective.

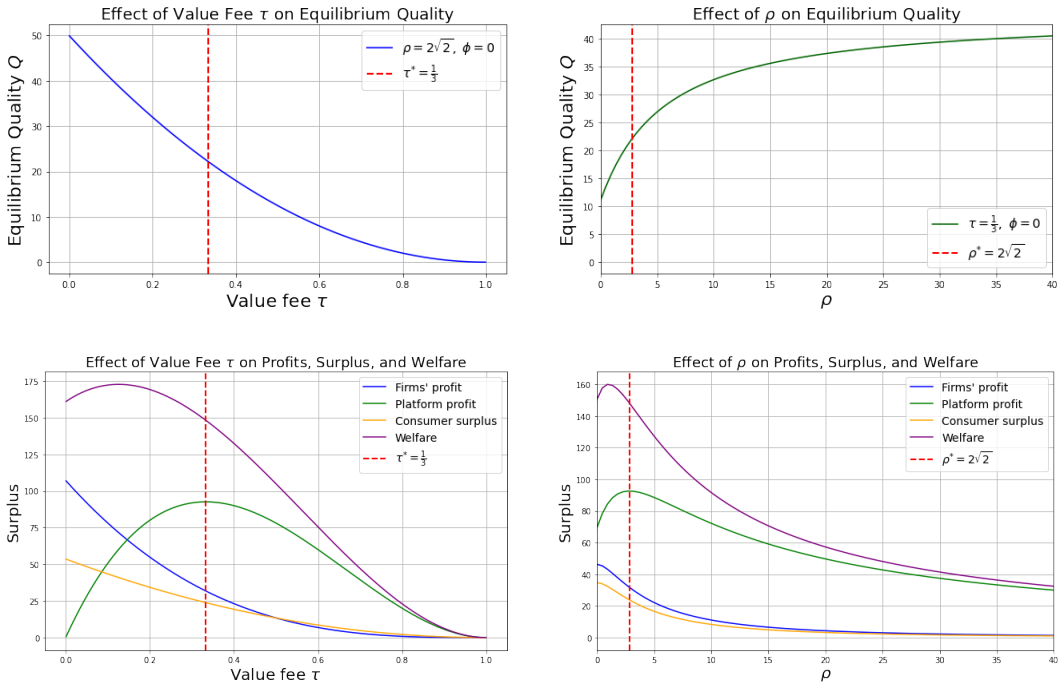


Figure 1: Numerical Example. Equilibrium quality and surpluses as functions of the value fee τ and the quality-reward parameter ρ, holding the other parameters at their equilibrium values.

The proposition shows that the choices by the platform are characterized by excessive quality rewards and, under some qualification that is met for instance in the numerical example above, excessive value fees. This analysis keeps all other parameters fixed and shows local welfare effects. Policy interventions, such as commission caps, are not covered by this analysis as they would lead to a constrained optimization problem, in which the platform can re-optimize over all parameters. This is studied next.

3.3 Commission Caps

Suppose a policy intervention puts an upper bound $\bar{\tau} \geq 0$ on the value fee. This modifies Problem $\mathcal{P}$ by adding the constraint $\tau \leq \bar{\tau}$. Call the resulting problem $\mathcal{P}_{\bar{\tau}}$.

Proposition 3. *Suppose* $E(Q) := \frac{C'(Q)B(Q)}{B'(Q)C(Q)}$ *is non-decreasing on* $(0, \hat{Q})$. *Then for any commission cap* $\bar{\tau}$, *the platform's problem* $\mathcal{P}_{\bar{\tau}}$ *has a unique solution. Denote the capped-optimal choices by* $(\rho_c, \tau_c, \phi_c, Q_c)$ *and define* $\widetilde{\tau} := 1 - \sqrt{\frac{C(\hat{Q})}{B(\hat{Q})}}$, *observing* $0 < \widetilde{\tau} < \tau^*$ *for the optimal choice* τ^* *in the unconstrained problem* $\mathcal{P}$. *The solution to* $\mathcal{P}_{\bar{\tau}}$ *satisfies:*

(i) If $\bar{\tau} \geq \tau^*$, *the solution is the same as in Theorem 1:* $(\rho_c, \tau_c, \phi_c) = (\rho^*, \tau^*, 0)$ *and* $Q_c = Q^*$.

(ii) If $\widetilde{\tau} \leq \bar{\tau} < \tau^*$*, then* $\rho_c > 0$*,* $\tau_c = \bar{\tau}$ *and* $\phi_c = 0$*, and* $Q^* < Q_c \leq \hat{Q}$*.*

(iii) If $0 \leq \bar{\tau} < \widetilde{\tau}$*, then* $\tau_c = \bar{\tau}$*,* $\phi_c > 0$*,* $Q_c = \hat{Q}$*, and* $\rho_c > 0$ *if and only if* $\bar{\tau} > 0$*.*

Social welfare is strictly increasing in the cap on $[0, \widetilde{\tau}]$*, strictly decreasing on* $[\widetilde{\tau}, \tau^*]$*, and constant for* $\bar{\tau} \geq \tau^*$*.*

In regime (i) the additional constraint does not affect the optimal solution. Hence, the the discussion of the unconstrained platform-optimal design applies.

In regime (ii) the constraint binds and reduces the chosen commission below its unconstrained optimum. Still, the platform generates revenues without using the transaction fee and quality is under-provided. This holds until the regime boundary, where $\bar{\tau} = \tilde{\tau}$ and $Q_c = \hat{Q}$. After that point, i.e., in regime (iii), there is a qualitatively new platform behavior. The commission cap is still binding, but besides the value fee, the platform uses the transaction fee. In regime (iii), the quality reward is strictly positive if and only if the commission cap is positive. The platform adjusts steering so that the additional incentive from consumer awareness exactly offsets the commission-induced distortion of quality incentives. Consequently, firms choose first-best quality.

Interestingly, social welfare is non-monotonic in the cap and is maximized at $\bar{\tau} = \widetilde{\tau}$. High caps do not affect the outcome as they are non-binding. Throughout the moderately binding regime, tighter caps move quality closer to the first best and increase welfare. Below $\widetilde{\tau}$, however, the platform substitutes transaction fees and reduces steering. Quality remains efficient, but quantity and welfare decline.

Hence, Proposition 3 shows that there is a unique welfare-maximizing commission cap $\bar{\tau} = \widetilde{\tau} = 1 - \sqrt{\frac{C(\hat{Q})}{B(\hat{Q})}}$, which is smaller than the unconstrained value fee τ^*. In fact, at this point the two cases (ii) and (iii) coincide and first-best quality $\hat{Q}$ is obtained. Proposition 3 can also be illustrated with our example introduced above.

Example. *Let, again,* $C(Q) = Q$ *and* $B(Q) = 10\sqrt{Q}$ *and, where needed,* $n = 2$*. The commission-cap threshold is* $\widetilde{\tau} = 1 - \sqrt{\frac{C(\hat{Q})}{B(\hat{Q})}} = 1 - \frac{1}{\sqrt{2}} \approx 0.2929$*. Caps at or above* $\tau^* = 1/3$ *leave the equilibrium unchanged. For* $\widetilde{\tau} \leq \bar{\tau} < 1/3$*, the platform charges* $\tau_c = \bar{\tau}$ *and no transaction fee, while* $Q_c = 50(1 - \bar{\tau})^2$*, and* $\rho_c = 2\sqrt{2}$*. Thus, a moderately binding cap increases quality and quantity. At* $\bar{\tau} = \widetilde{\tau}$*, quality reaches* $\hat{Q} = 25$ *and welfare attains its maximum over commission caps,* $W = 156.25$*.*

For stricter caps, quality remains at $\hat{Q} = 25$*, but the platform introduces a transaction fee. Further tightening therefore reduces quantity and welfare. At a zero cap, the platform chooses* $\rho_c = 0$ *and* $\phi_c = 12.5$*, reproducing the transaction-fee-only scenario in Table 1. Welfare then falls to* 136.72*, below its unconstrained equilibrium level, despite first-best quality.*

Table 2 summarizes the outcomes and each group's surplus for several caps. (The first row repeats what we have seen in Table 1 for the case that only transaction fees are allowed. The last row repeats what we have seen there as the unconstrained equilibrium values.) Even the welfare-maximizing commission cap does not achieve first-best welfare: quality is efficient, but quantity remains distorted. The zero-fee monopoly benchmark likewise attains efficient quality, while monopoly pricing reduces welfare relative to the first best.

Table 2: Numerical example: equilibrium outcomes under different commission caps.

	Quality Q	Welfare	Platform profit	Consumer surplus	Firm profit	Price p	Quantity y
Zero commission cap: $\rho = 0$, $\tau = \bar{\tau} \equiv 0$, $\phi = 12.5$	25.00	136.72	78.13	19.53	39.06	43.75	6.25
Cap below welfare-maximizing cap: $\rho \approx 1.3059$, $\tau = \bar{\tau} \equiv 0.15$, $\phi \approx 6.0135$	25.00	146.09	84.46	22.83	38.81	43.24	6.76
Welfare-maximizing cap: $\rho = 2\sqrt{2}$, $\tau = \bar{\tau} \equiv 1 - 1/\sqrt{2} \approx 0.2929$, $\phi = 0$	25.00	156.25	91.53	26.81	37.91	42.68	7.32
Cap above welfare-maximizing cap: $\rho = 2\sqrt{2}$, $\tau = \bar{\tau} \equiv 0.32$, $\phi = 0$	23.12	150.99	92.48	24.79	33.72	41.04	7.04
Nonbinding cap: $\bar{\tau} \geq \frac{1}{3}$ $\rho = 2\sqrt{2}$, $\tau = \frac{1}{3}$, $\phi = 0$	22.22	148.19	92.59	23.83	31.77	40.24	6.90

Figure 2 illustrates the result. Social welfare is non-monotonic in the commission cap, starting at a low level when $\bar{\tau} \equiv 0$, as the transaction fee then used induces the double marginalization problem and there is no steering. When the tight commission cap is loosened, the value fee together with steering is used to substitute the transaction fee. Welfare then increases in the cap, while keeping quality at the first best level $\hat{Q} = 25$ up to its peak, $\bar{\tau} = \widetilde{\tau}$. After that threshold, throughout the moderately binding regime, steering remains constant at $\rho_c = 2\sqrt{2}$. (So the outcomes coincide with those obtained by reducing the value fee while holding steering fixed.)

4 Extensions and Robustness

4.1 Quality investments

In our model, the costs of higher quality accrue for each unit that is produced. Indeed, the marginal cost of production $C(Q_i)$ is increasing in quality. Depending on the technology, there can additionally be bulk investments into quality, which improve the quality of each product. These costs, say $K(Q_i)$, are increasing in quality but fixed with respect to quantity. This changes a firm's profit function, and

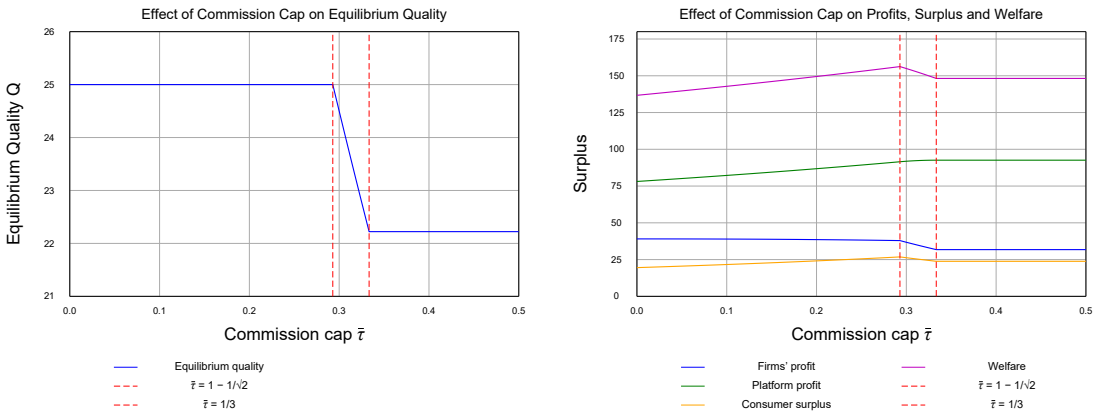


Figure 2: Numerical example: equilibrium outcomes under different commission caps $\bar{\tau}$. The dashed lines mark regime shifts at $\widetilde{\tau} = 1 - 1/\sqrt{2}$ and $\tau^* = 1/3$. Social welfare is non-monotonic in the commission cap, starting at a low level when $\bar{\tau} \equiv 0$, as the transaction fee then used induces the double marginalization problem. Welfare then increases, while keeping quality at the first best level up to its peak. After that threshold, commissions are excessive and reduce welfare. Finally, when commission caps are higher than the unconstrained value fee, they are not binding and the unconstrained equilibrium remains.

its maximization problem becomes

$$\max_{Q_i, y_i} \qquad \tilde{\pi}_i = A(Q_i, Q_{-i}) \cdot y_i \cdot [P(y_i, Q_i)(1-\tau) - C(Q_i) - \phi] - K(Q_i),$$

where $C(Q_i)$ is again the marginal cost of production; and $K(Q_i)$ is the investment cost, with $K'(Q_i) > 0$ and $K(0) = 0$.

The costs $K(Q_i)$ here are similar to the "effort" costs in a standard Tullock contest. The benefits, however, do not only capture the increase in probability of winning ($A(Q_i, Q_{-i})$), but also affect the "prize," as inverse demand and marginal costs depend on quality Q_i.

We focus again on symmetric equilibria and study the two first-order conditions that led to Proposition 1. While the quantity first-order condition (2) is unchanged because investment costs do not enter quantity optimization, the quality first-order condition (3) now becomes

$$\frac{\rho \sum_{j \neq i} Q_j^\rho}{Q_i(Q_i^\rho + \sum_{j \neq i} Q_j^\rho)} = \frac{C'(Q_i) - B'(Q_i)(1-\tau) + \dfrac{K'(Q_i)}{A(Q_i, Q_{-i}) y_i}}{(B(Q_i) - y_i)(1-\tau) - C(Q_i) - \phi}. \tag{5}$$

Combining them under symmetry yields

$$\frac{C'(Q) - B'(Q)(1-\tau) + \dfrac{nK'(Q)}{y}}{B(Q)(1-\tau) - C(Q) - \phi} = \frac{\rho(n-1)}{2nQ}, \tag{6}$$

where $y = \frac{B(Q)(1-\tau)-C(Q)-\phi}{2(1-\tau)}$. This differs from (4) in Proposition 1 by the term $nK'(Q)/y$, and it is only a necessary condition for an interior symmetric equilibrium here. In it, quantity is unchanged conditional on quality, and maintaining the same quality at given fees requires a higher ρ when $K'(Q) > 0$.

For $\rho = 0$, firms would choose an interior quality such that the LHS of (6) is zero:

$$(1-\tau)B'(Q) - C'(Q) = \frac{nK'(Q)}{y}.$$

As $K'(Q) > 0$, this is obtained at a quality below the level $\tilde{Q}$ that is chosen without investment costs. Intuitively, the additional costs of quality drive equilibrium quality down.

However, also the first-best quality is affected by quality investment costs and a symmetric solution is no longer socially optimal.

4.2 More general demand functions

So far we have assumed quasi-linear demand. This choice is very convenient because it makes the change in demand due to quality changes independent of quantity. Formally, with quasi-linear demand we have $\frac{\partial P(y,Q)}{\partial Q} = B'(Q)$ and hence $\frac{\partial^2 P(y,Q)}{\partial y \partial Q} = 0$. In general, this need not be the case. Spence (1975) shows that the cross-derivative effects lead to a downwards distortion of quality for $\frac{\partial^2 P(y,Q)}{\partial y \partial Q} < 0$ and an upwards distortion for $\frac{\partial^2 P(y,Q)}{\partial y \partial Q} > 0$. The reason is that the monopolistic firm adjusts its quality choice to the marginal consumer, not to the average consumer.

This Spence distortion comes on top of the two distortions that we investigate in this paper (customer awareness reward for quality and value fee) without changing their substance. In particular, if the marginal consumer's willingness-to-pay for additional quality is low relative to the average consumer's (i.e., $\frac{\partial^2 P(y,Q)}{\partial y \partial Q} < 0$), then quality is further lowered by optimizing firms.

4.3 More general consideration sets

A literal interpretation of our model was that each consumer considers only one product and their demand (given awareness) hence depends only on this product's price and quality. More realistically, most consumers consider a non-singleton subset

of all products. Such an extension would not affect our analysis under the additional assumption that products are *totally differentiated*.

Allowing for (partial) substitutability of different products and non-singleton consideration sets, generates firm competition on two levels. Firms compete for customers' limited attention to become part of their considerations sets; and they compete against the other products within any consideration sets.

This can be modeled following the framework of Dinerstein et al. (2018). Let $a_L(\mathbf{p}, \mathbf{Q})$ be the probability that a consumer faces consideration set $L \subseteq N$ for the profile of prices $\mathbf{p}$ and the profile of qualities $\mathbf{Q}$. Let $\tilde{D}_i(\mathbf{p}, \mathbf{Q}|L)$ be the demand for good i when a consumer considers the firms in L. Then expected demand is $\sum_{L \in 2^N} a_L(\mathbf{p}, \mathbf{Q}) \cdot \tilde{D}_i(\mathbf{p}, \mathbf{Q})|L)$. Our key assumption is that *expected demand is separable into probability of awareness and demand given awareness*: $\sum_{L \in 2^N} a_L(\mathbf{p}, \mathbf{Q}) \cdot \tilde{D}_i(\mathbf{p}, \mathbf{Q})|L) = A_i(\mathbf{p}, \mathbf{Q}) \cdot D_i(\mathbf{p}, \mathbf{Q})$ for an algorithm A_i and (average) demand function D_i.[7,8] Hence, a re-interpretation of our demand function is that it covers *the average demand a firm faces*. The average is build over all possible considerations sets with averaging weights according to the probability that each set is obtained. In such a setting each firm chooses the best response to all other firms' choices and hence sets the quality according to the same incentives as discussed above. Indeed, stronger competition by closely substitutable products are mimicked by a highly elastic demand in our model, where firms act as monopolists. However, the model that we analyze is a simplified version in which all firms are the same. This is for tractability, as it allows us to use the symmetric solution in the game between firms in order to focus on the platform's problem.

5 Discussion

We develop a simple model to investigate how firms' quality choices depend on platform design. The profit-maximizing platform rewards quality by giving higher-quality products additional consumer attention. At the same time, it charges a value fee that weakens quality incentives. Together, these choices induce firms to under-provide quality in equilibrium. The commission—equal to one third in our numerical example—therefore provides a starting point for considering policy inter-

[7]This separation is also possible in Dinerstein et al. (2018)'s framework. Their specification of demand for a product j, given by their equations (1)-(3), can be rewritten as $\sum_{L \in 2^L} (\prod_{i \in L} a_i)(\prod_{i \notin L}(1 - a_i))D_j(p_j, p_{-j}|L) = a_j \cdot \sum_{L \in 2^L : j \in L} (\prod_{i \in L \setminus j} a_i)(\prod_{i \notin L}(1 - a_i))D_j(p_j, p_{-j}|L)$. Hence, it is separable into a probability of considering product j, here called a_j and the remainder part, which can be interpreted as an average demand D_j.

[8]One implication of this independence assumption is that it does not allow platforms to produce compositions of products based on their substitutability. Maybe a platform would have incentives to present rather differentiated goods, an extreme case of which is treated by our model.

ventions. However, welfare cannot be assessed from quality alone. A small reduction in the quality-reward parameter from its platform-optimal value, holding the other parameters fixed, increases quantity and social welfare, even though it moves quality further below its efficient level. In this sense, the quality incentive is set too high from a welfare perspective. The platform values the higher prices induced by quality competition because it receives a share of sales revenue.

Commission caps allow us to examine regulation when the platform adjusts its remaining instruments. Tightening a moderately binding cap increases quality, quantity, and welfare. Once quality reaches its efficient level, further tightening induces the platform to introduce a positive transaction fee. Quality remains efficient, but quantity and welfare fall. Thus, the welfare-maximizing cap achieves efficient quality without implementing the first-best quantity. In our numerical example, a zero commission cap yields lower welfare than the unconstrained platform optimum because the platform substitutes toward transaction fees and induces low quantities.

We keep the model simple to isolate the interaction between fees and quality incentives. Several assumptions delimit the scope of our results. First, the platform allocates consumer awareness according to relative quality, without personalizing recommendations or directly rewarding low prices or high quantities. Personalization would allow the platform to match products to consumers with different preferences, while price- or quantity-based recommendations would provide additional instruments for influencing firms' choices. These possibilities could change both quality incentives and the preferred fee structure. Our analysis characterizes the platform-optimal algorithm within the specified functional form, rather than among all possible recommendation rules.[9]

Second, marginal production costs $C(Q)$ depend on quality but not on output. With output-dependent marginal costs, additional consumer awareness would also change the cost of serving an additional customer. This would introduce a further interaction between the allocation of attention, pricing, and quality. Whether our optimal-design and welfare results extend to this setting remains an open question.

Third, firms are homogeneous and their number is fixed. Under these assumptions, we establish that the equilibrium is unique and symmetric whenever trade is positive. Heterogeneous firms could differ in quality, visibility, and sales, giving the platform an additional incentive to steer consumers toward particular suppliers. Endogenous entry would introduce a further trade-off: fees and recommendation rules would affect not only incumbent firms' choices but also participation. Both extensions would require accounting for changes in market composition when evaluating platform design and regulation.

[9]For a continuous set-up, a related question is addressed in Valvassori Bolgè (2026).

Finally, quality is observed by both the platform and consumers. The inefficiencies we identify therefore arise even without asymmetric information about quality. When quality is imperfectly observed, recommendations may also convey information, adding an informational role to the incentive effects studied here.

A Proofs

This is a short, but still self-contained, version of the proofs.[10] Throughout the appendix, write

$$a := 1 - \tau, \quad V(Q) := aB(Q) - C(Q) - \phi, \quad S(Q) := B(Q) - C(Q),$$

$$z(Q) := \frac{C(Q)}{B(Q)}, \quad r(Q) := \frac{C'(Q)}{B'(Q)}.$$

For $Q > 0$, convexity and strict concavity give $C \leq QC'$ and $B > QB'$, hence $0 < z < r$. Moreover, $z' = (B'/B)(r - z) > 0$ and $r' > 0$.

A.1 Proof of Proposition 1

If $\tau \geq 1$, every feasible positive sale has a negative margin. If $a > 0$ but $\max_{Q \geq 0} V(Q) \leq 0$, the same conclusion follows from $\pi_i = A_i y_i [V(Q_i) - a y_i]$. In either case, precisely the profiles with all quantities zero are equilibria.

Otherwise, strict concavity gives a unique maximizer $\tilde{Q} > 0$ and an interval $(\underline{Q}, \overline{Q})$ on which $V > 0$. A firm can always earn positive profit by choosing quality $\tilde{Q}$ and a sufficiently small positive quantity. Thus every equilibrium is interior. Optimizing quantity yields

$$y_i = \frac{V(Q_i)}{2a}, \qquad g_i(Q_i) := \max_{y_i} \pi_i = \frac{A_i V(Q_i)^2}{4a} \quad (V(Q_i) > 0). \tag{A.1}$$

Set $R := -V'/V$. Then $R' > 0$, $R(\tilde{Q}) = 0$, and $R(Q) \to \infty$ as $Q \uparrow \overline{Q}$. For $\rho > 0$ and $T_i := \sum_{j \neq i} Q_j^\rho > 0$,

$$\frac{g_i'}{g_i} = \frac{\rho T_i}{Q_i(Q_i^\rho + T_i)} - 2R(Q_i). \tag{A.2}$$

The first term is positive and strictly decreasing; the second is strictly increasing. Their difference is positive up to $\tilde{Q}$ and crosses zero exactly once thereafter. This proves global optimality and uniqueness of the best response. If $\rho = 0$ or all rivals have zero quality, the unique best-response quality is simply $\tilde{Q}$.

For $\rho > 0$, every equilibrium has positive qualities. With $T := \sum_j Q_j^\rho$, its first-order conditions read

$$2Q_i R(Q_i) = \rho \left(1 - \frac{Q_i^\rho}{T}\right).$$

[10] A more extensive version of the proofs with additional explanations can be found here: `www.berno.info`.

All $Q_i > \tilde{Q}$; on this interval the left side strictly increases in Q_i and the right side strictly decreases. Holding the common T fixed therefore forces all qualities to coincide. Under symmetry the condition is

$$QR(Q) = \frac{\rho(n-1)}{2n}. \tag{A.3}$$

Its left side increases continuously from zero to infinity on $[\tilde{Q}, \overline{Q})$, giving exactly one solution. The global best-response argument verifies that this solution is an equilibrium. For $\rho = 0$, all firms choose $\tilde{Q}$ directly. Equation (A.1) then determines the unique quantity in either case. □

A.2 Proof of Theorem 1

Proposition 1 shows that (Q, τ, ϕ) is implementable with positive trade exactly when $Q > 0$, $a > 0$, $V(Q) > 0$, and $r(Q) \geq a$. The implementing reward is

$$\rho = \frac{2nQ}{n-1} \frac{C'(Q) - aB'(Q)}{V(Q)}, \qquad y = \frac{V(Q)}{2a}. \tag{A.4}$$

Eliminating the transaction fee gives platform profit $\Pi^{\mathrm{Pl}} = S(Q)y - (1+a)y^2$. If $\phi > 0$, replace a by $a' := aC/(C+\phi) < a$ and set $\phi' = 0$. This preserves Q, y, remains implementable since $r \geq a > a'$, and raises profit by $(a - a')y^2$. Hence $\phi^* = 0$.

The reduced problem is therefore to maximize

$$F(Q, \tau) = \frac{\tau}{4}\left(B(Q)^2 - \frac{C(Q)^2}{a^2}\right), \qquad z(Q) < a \leq \min\{r(Q), 1\}. \tag{A.5}$$

A positive maximum exists. Indeed, $z < r$ provides feasible designs with positive profit. Positive trade requires $0 < Q < Q_{\max}$, where $B(Q_{\max}) = C(Q_{\max})$. The bounds $0 \leq F \leq B^2/4$ and $F \leq (1-z)B^2/4$ imply that profit tends to zero at either quality endpoint. A positive upper level set can thus be restricted to a compact interior quality interval; closing its trade boundary adds only zero-profit points. Continuity gives attainment. Every optimum has $0 < \tau < 1$ and positive trade.

The reward constraint is slack at an optimum: at $a = r(Q)$,

$$F_Q = \frac{\tau B'}{2}\left(B - \frac{C}{a}\right) > 0,$$

and increasing Q slightly preserves feasibility because $r' > 0$. Thus $\rho^* > 0$, and both

reduced first-order conditions hold:

$$rz = a^2, \qquad z^2 = \frac{a^3}{2-a} \quad \Longrightarrow \quad \tau^* = \sqrt{1 - r(Q^*)^2}. \tag{A.6}$$

In particular, $r(Q^*) < 1$, so $Q^* < \hat{Q}$.

Finally, with $E = r/z$, these conditions imply $a(1+E) = 2$ and

$$z(Q)\sqrt{E(Q)}[1 + E(Q)] = 2.$$

If E is nondecreasing below $\hat{Q}$, the left side is strictly increasing there because $z' > 0$. Consequently there is at most one optimal quality. Existence and the implementation formula then give a unique design. □

A.3 Proof of Proposition 2

Set $\phi = 0$, $G := C' - aB' > 0$, and $H := QG/V$ near the optimum. The equilibrium condition is $H = \rho(n-1)/(2n)$, and

$$H_Q = \frac{G + QG'}{V} + \frac{QG^2}{V^2} > 0, \qquad H_\tau = \frac{Q(B'V + BG)}{V^2} > 0.$$

Therefore $Q_\rho > 0$ and $Q_\tau < 0$. Since $y = V/(2a)$, $y_\rho = -GQ_\rho/(2a) < 0$. Holding ρ fixed instead gives

$$y_\tau = -\frac{C^2}{2a^2 V H_Q} \frac{d}{dQ}\left(\frac{QG}{C}\right).$$

Now $QG/C = \varepsilon_C - aQB'/C$, while

$$\left(\frac{QB'}{C}\right)' = \frac{B'}{C}\left(1 + \frac{QB''}{B'} - \varepsilon_C\right) < 0$$

because $\varepsilon_C \geq 1$ and $B'' < 0$. Thus $\varepsilon_C'(Q^*) \geq 0$ ensures $y_\tau|_* < 0$.

Consumer surplus is $y^2/2$ and aggregate firm profit is ay^2. Their derivatives have the asserted signs. At the optimum the platform's total profit derivative with respect to either interior instrument is zero. Consequently,

$$W_\rho|_* = (1+2a)yy_\rho|_* < 0, \qquad W_\tau|_* = (1+2a)yy_\tau|_* - y^2 < 0.$$

These strict derivatives establish all the stated local improvements. □

A.4 Proof of Proposition 3

Nonbinding caps preserve the unique unconstrained optimum. For a binding cap, write $a^* := 1 - \tau^*$, $\widetilde{a} := \sqrt{z(\hat{Q})}$, and $h(Q) := r(Q)z(Q)$. Both r, z increase, so $h' > 0$. Moreover, $(a^*)^2 = r(Q^*)z(Q^*) < z(Q^*) < z(\hat{Q})$, hence $a^* < \widetilde{a} < 1$.

Fix $a \in [a^*, 1]$. Implementation requires $0 < y \leq [aB(Q) - C(Q)]/(2a)$ and $r(Q) \geq a$. Maximizing $S(Q)y - (1+a)y^2$ over quantity yields

$$y(Q, a) = \min\left\{\frac{S(Q)}{2(1+a)}, \frac{aB(Q) - C(Q)}{2a}\right\},$$

and optimized profit

$$J(Q, a) = \begin{cases} S(Q)^2/[4(1+a)], & z(Q) \leq a^2, \\ (1-a)[B(Q)^2 - C(Q)^2/a^2]/4, & z(Q) \geq a^2. \end{cases}$$

The branches agree at $z = a^2$. Their quality derivatives have the signs of S' and $a^2 - h$, respectively (the second branch is absent from the positive-trade domain at $a = 1$).

For $a \in [a^*, \widetilde{a})$, let $Q(a)$ solve $h(Q) = a^2$. It lies in $[Q^*, \hat{Q})$, satisfies $z < a < r$ and $z > a^2$, and therefore belongs to the second branch. The first branch increases below $Q(a)$, while the second increases up to $Q(a)$ and decreases thereafter. Thus $Q(a)$ is the unique global maximizer and has $\phi = 0$, $\rho > 0$. For $a \in [\widetilde{a}, 1]$, the first branch is maximized at $\hat{Q}$; the second, if present, lies above $\hat{Q}$ and decreases. Hence the unique optimum has

$$Q = \hat{Q}, \qquad y = \frac{S(\hat{Q})}{2(1+a)}, \qquad \phi = \frac{a^2 B(\hat{Q}) - C(\hat{Q})}{1+a}. \tag{A.7}$$

These allocations are feasible, and the two regimes coincide at $\widetilde{a}$.

It remains to optimize the commission. For $a \in [a^*, \widetilde{a})$, the envelope theorem gives

$$\frac{d}{da}\max_Q J(Q, a) = \frac{B(Q(a))^2}{4}\left[\frac{2-a}{aE(Q(a))} - 1\right].$$

At a^*, $a^*(1 + E(Q^*)) = 2$. Since $Q(a)$ increases and E is nondecreasing, this derivative is strictly negative for $a > a^*$. For $a \geq \widetilde{a}$, optimized profit is $S(\hat{Q})^2/[4(1+a)]$, also strictly decreasing. The cap therefore binds: $a = 1 - \bar{\tau}$. Positive profits are attainable even at a zero cap, so no-trade designs cannot be optimal. Equation (A.4)

uniquely determines steering; in the tight regime it gives

$$\rho_c = \frac{2n}{n-1}\frac{\hat{Q}B'(\hat{Q})(1-a^2)}{aS(\hat{Q})}.$$

This proves the three design regimes, including the zero-cap boundary.

Finally, $W = S(Q)y - y^2/2$. In the moderate regime, $Q_{\bar{\tau}} = -2a/h' < 0$. Using $E' \geq 0$ gives $h' = r'z + rz' \geq 2rz' = 2r(B'/B)(r-z)$, and hence

$$y_{\bar{\tau}} = -\frac{C}{2a^2} + \frac{C' - aB'}{h'} \leq \frac{B}{2r}\frac{z-a}{r-z} < 0.$$

Since $S' \geq 0$ and $S - y > 0$, welfare strictly decreases with the cap in this regime. In the tight regime, $Q = \hat{Q}$ and $y = S(\hat{Q})/[2(2-\bar{\tau})]$ strictly increases with the cap; therefore $W_{\bar{\tau}} = (S-y)y_{\bar{\tau}} > 0$. The allocations are continuous at the boundaries and constant for nonbinding caps. Thus welfare has the claimed non-monotonic shape and unique maximum at $\bar{\tau} = \widetilde{\tau}$. □

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